

# Gravitating global defects: the gravitational field and compactification

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## Abstract

We give a prescription to add the gravitational field of a global topological defect to a solution of Einstein's equations in an arbitrary number of dimensions. We only demand that the original solution has a  $O(n)$  invariance with  $n \geq 3$ . We will see that the general effect of a global defect is to introduce a deficit solid angle. We also show how the same kind of scalar field configurations can be used for spontaneous compactification of  $n$  extra dimensions on an  $n$ -sphere.

## I. INTRODUCTION

Global defects arise in theories in which a global symmetry is broken and the vacuum manifold is nontrivial [1]. An example where this kind of topological defect can arise is given by a theory with  $n$  scalar fields and a potential which has a vacuum manifold with the topology of a  $(n - 1)$ -sphere. This is the case we will consider here.

Global defects were first considered in four spacetime dimensions, but more recently they have also been studied in the context of the brane-world scenario. In four spacetime dimensions, when  $n = 2$ , the defects are global strings [2] and when  $n = 3$ , global monopoles [3]. In the brane-world scenarios one models our 'visible' universe as a 3-brane living in a higher-dimensional spacetime with  $n$  extra dimensions. In this setup one can imagine the 3-brane being the core of a global defect. Such a possibility has been investigated in a number of papers. The domain wall case, a single extra dimension, was studied long ago in Ref. [4]. Explicit geometries in higher dimensions are more recent. The case with  $n = 2$ , two extra dimensions, has been considered in [5] and [6]. In [7] solutions to Einstein's field equations (EFE's) with  $n > 2$ , three and higher extra dimensions were given, in [8] the localization of the graviton in some of those solutions was discussed, while in Ref. [9] Oda addressed the issue of localization of various fields.

In the present paper we want to study further the gravitational effects of global defects for  $n \geq 3$ . We will show how to construct new solutions to Einstein's equations from known ones. We will see that the net effect of the addition of a global charge, away from the core, is to introduce a deficit solid angle, just as it was shown for the global monopole,  $n = 3$ , in four dimensions by Barriola and Vilenkin [3].

We will also show that higher-dimensional solutions with geometry  $\mathcal{M}^q \times S^{n-1}$  can be constructed, where  $\mathcal{M}^q$  is a solution to  $q$ -dimensional Einstein gravity.

The plan of the paper is as follows. In the next section we introduce the class of global defect that we are going to study. In section III we show how to add the gravitational field of the global defect to a known solution of Einstein's equations. In section IV we use the global defect for compactification on a  $(n - 1)$ -sphere. Conclusions are summarized in section V.

## II. GLOBAL DEFECTS AND THE $\sigma$ -MODEL APPROXIMATION

In this section we present the class of models that we have in mind.

To keep the discussion as general as possible, in this section we consider spacetime to be  $d$ -dimensional with a metric of the form

$$ds^2 = g_{AB}dx^A dx^B = g_{ij}(x^i)dx^i dx^j + B^2(x^i)d\Omega_{n-1}^2 \quad (1)$$

where  $d\Omega_{n-1}^2$  is the metric on a  $(n - 1)$ -sphere of unit radius which is parametrized by the  $n - 1$  angular coordinates  $y^\alpha$ .

The models that we are interested in include a multiplet of  $n$  scalar fields  $\phi^a$ ,  $a = 1 \dots n$  with a potential  $V(\phi^a \phi^a)$  that has a minimum on the  $(n - 1)$ -sphere  $\phi^a \phi^a = \eta^2$ . We further assume that  $V(\phi^a \phi^a = \eta^2) = 0$ , since a nonzero value can be absorbed into the cosmological constant term in Einstein's equations.

It is well known that if the mapping of a  $(n - 1)$ -sphere in configuration space in the vacuum manifold is non-trivial, continuity requires the fields to take values outside the vacuum manifold in some region surrounding the origin. This region is called the core of the defect. At a distance from the core, where the fields are in the vacuum manifold, the lagrangian is that for  $n$  scalar fields subject to the constraint  $\phi^a \phi^a = \eta^2$ . Outside the core the system is thus well approximated by a nonlinear  $\sigma$ -model, and the equations for the fields can be written as

$$\nabla^2 \phi^a + \eta^{-2} (\partial_A \phi^b \partial^A \phi^b) \phi^a = 0 \quad (2)$$

We shall consider the simplest non-trivial *ansatz*, the familiar hedgehog configuration, in which the fields depend only on the angular variables  $y^\alpha$  through the relation  $\vec{\phi}(\hat{r}) = \eta \hat{r}$ , where  $\hat{r}$  is the unit vector on the  $(n - 1)$ -sphere. It is not hard to check that this configuration does indeed satisfy the field equations.

To solve Einstein's equations we need the expression for the energy momentum tensor. For the hedgehog configuration it is

$$T_j^i = -\frac{1}{2}(n-1)\frac{\eta^2}{B^2(x^i)} \delta_j^i, \quad T_\beta^\alpha = -\frac{1}{2}(n-3)\frac{\eta^2}{B^2(x^i)} \delta_\beta^\alpha. \quad (3)$$

It is apparent from this expressions that if  $B^2(x^i)$  goes to zero at some point, the energy-momentum tensor is singular. This signals the presence of the defect core where continuity requires that fields go to zero, which in turn removes the singularity of the energy-momentum tensor. We see how the presence of the defect is related to the fact that the metric function  $B^2(x^i)$  vanishes at some point. Accordingly, if  $B^2(x^i)$  does not vanish at any point, the fields can stay in the vacuum manifold over the whole space. If this is the case, no defect is present and the  $\sigma$ -model is exactly applicable over the entire space. An example is any geometry of the form  $\mathcal{M}^{(d-n+1)} \times S^{n-1}$  in which  $n - 1$  dimensions are compactified on a sphere. We shall consider both cases in the following sections.

### III. ADDING A GLOBAL DEFECT

In this section we are going to show that just as the effect of local strings is to introduce a deficit angle in the plane transverse to the string, the effect of a global defect in the hedgehog configuration is to introduce a deficit solid angle. We will also give a prescription to construct solutions to Einstein's equations with global topological charges in the  $\sigma$ -model approximation described above.

We start by considering a solution to Einstein's equations in  $d = q + n - 1$  dimensions in the following form:

$$ds_0^2 = g_{ij}(x^i)dx^i dx^j + A^2(x^i)d\Omega_{n-1}^2 \quad (4)$$

$$d\Omega_{n-1}^2 = \hat{g}_{\alpha\beta}(y^\beta)dy^\alpha dy^\beta \quad (5)$$

with  $i, j = 1..q$  and  $\alpha, \beta = 1..n - 1$ . The  $\{y^\alpha\}$  are the angular coordinates in  $d\Omega_{n-1}^2$ . This is a general example of a warped geometry with warp factor  $A^2(x^i)$ . For such geometries the Ricci tensor splits as

$$R_{ij}^0 = \tilde{R}_{ij} - (n-1) \frac{A_{;ij}}{A} \quad (6)$$

$$R_{\alpha\beta}^0 = \hat{R}_{\alpha\beta} - \hat{g}_{\alpha\beta} [A \square A + (n-2)\tilde{\nabla}A\tilde{\nabla}A] \quad (7)$$

The 0 superscripts refer to quantities for the metric (4). Tildes stand for quantities calculated with the  $q$ -dimensional metric  $g_{ij}(x^i)dx^i dx^j$  and the hats refer to quantities for  $d\Omega_{n-1}^2$ .

We assume that the above is a solution to EFE's with a stress-energy tensor  $T_{AB}^0$ ,

$$R_{AB}^0 - \frac{1}{2} g_{AB}^0 R^0 = \kappa^2 T_{AB}^0 \quad (8)$$

We will now modify the original metric by multiplying the warp factor by a constant factor  $(1 - \Delta)$ . The line element thus obtained is

$$ds^2 = g_{ij}(x^i)dx^i dx^j + (1 - \Delta)A^2(x^i)d\Omega_{n-1}^2 \quad (9)$$

This will be a solution to EFE's for a different energy-momentum tensor  $T_{AB} = \kappa^{-2}G_{AB}$  which we want to relate to  $T_{AB}^0$ .

We can use equations (6) and (7) to write the Ricci tensor of the new metric  $R_{AB}$  in terms of  $R_{AB}^0$

$$R_{ij} = R_{ij}^0 \quad (10)$$

$$R_{\alpha\beta} = \Delta \hat{R}_{\alpha\beta} + (1 - \Delta)R_{\alpha\beta}^0 \quad (11)$$

$$R = R^0 + \frac{\Delta}{1 - \Delta} \frac{\hat{R}}{A^2} \quad (12)$$

With these relations we construct the Einstein tensor from which we can directly read off  $T_{AB}$ . We find that

$$T_j^i = (T^0)_j^i - \frac{1}{2} \delta_j^i \frac{\Delta}{\kappa^2(1-\Delta)} \frac{\hat{R}}{A^2}, \quad (13)$$

$$T_\beta^\alpha = (T^0)_\beta^\alpha + \frac{\Delta}{\kappa^2(1-\Delta)} \frac{\hat{G}_\beta^\alpha}{A^2}. \quad (14)$$

Since the hats refer to  $d\Omega_{n-1}^2$ , we have  $\hat{R} = (n-1)(n-2)$  and  $\hat{G}_{\alpha\beta} = -(n-3)(n-2)\hat{g}_{\alpha\beta}/2$ . Putting these expressions into the equations above, we see that the effect of introducing the  $(1-\Delta)$  factor in the metric (4) corresponds to the addition of matter with energy-momentum tensor given by

$$T_j^i = -\frac{1}{2} \delta_j^i \frac{\Delta(n-2)}{\kappa^2(1-\Delta)} \frac{n-1}{A^2} \quad (15)$$

$$T_\beta^\alpha = -\frac{1}{2} \delta_\beta^\alpha \frac{\Delta(n-2)}{\kappa^2(1-\Delta)} \frac{n-3}{A^2} \quad (16)$$

It is clear that this is equivalent to (3) with  $B^2(x^i) = (1-\Delta)A^2(x^i)$  when we choose  $\Delta$  to have the value:

$$\Delta = \frac{\kappa^2 \eta^2}{n-2} \quad (17)$$

We thus have a prescription to add the gravitational field of a topological global charge to a known solution with  $O(n)$  invariance: we just have to multiply the  $d\Omega_{n-1}^2$  piece of the line element by the factor  $1 - (\kappa^2 \eta^2 / (n-2))$ , which we can interpret as a deficit solid angle.

We now apply the prescription described above to some examples.

For the usual four-dimensional spacetime the result presented above is applicable to every isotropic solution to Einstein's equations. Straightforward examples are the FRW cosmological models and the Schwarzschild and Reissner-Nordstrom black hole solutions.

The metric for the global monopole in a general FRW spacetime will look like

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1+kr^2} + (1-\kappa^2 \eta^2)r^2 d\Omega_2^2 \right\} \quad (18)$$

For the Schwarzschild and Reissner-Nordstrom solutions the metric, after the inclusion of the global charge, has the form

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + (1-\kappa^2 \eta^2)r^2 d\Omega_2^2 \quad (19)$$

The first thing we must note is that, due to the deficit solid angle, the metric is no longer asymptotically flat and so the no-hair theorem for black holes does not apply in this case. The metric for the Schwarzschild solution with a global topological charge was first derived by Barriola and Vilenkin in [3]. As they point out, such a topologically charged black hole could arise after the collision of a global monopole and a black hole. An analysis along the lines of [10], where the solution describing a black hole with a local cosmic string passing through it was given, shows that the area law for the black hole entropy still holds.

A global charge can also be added to black  $p$ -branes [11]

$$ds^2 = e^{2A}(-e^{2f}dt^2 + d\vec{x}^2) + e^{2B} \left[ e^{-2f}dr^2 + r^2 \left(1 - \frac{\kappa^2 \eta^2}{n-2}\right) d\Omega_{n-1}^2\right]. \quad (20)$$

See Ref. [11] for details about the metric functions  $A$ ,  $B$  and  $f$  as well as the bulk gauge and scalar fields also present. Likewise we can add the far field of a global defect in any brane in the same way, as long as the metric induced on the brane has the required  $O(n)$  symmetry.

#### IV. COMPACTIFICATION

In this section we show that it is possible to construct new compactified solutions from lower dimensional ones by adding appropriate scalar fields. In this case we start with a general  $q$ -dimensional metric  $ds_0^2$  and consider the higher dimensional metric

$$ds^2 = ds_0^2 + C^2 d\Omega_{n-1}^2, \quad (21)$$

$$ds_0^2 = g_{ij}(x^i) dx^i dx^j, \quad (22)$$

$$C^2 d\Omega_{n-1}^2 \equiv C^2 \hat{g}_{\alpha\beta} dy^\alpha dy^\beta \equiv g_{\alpha\beta} dy^\alpha dy^\beta, \quad (23)$$

$$i, j = 1 \dots q, \quad \alpha, \beta = 1 \dots n-1, \quad (24)$$

where  $C^2$  is a constant and  $\hat{g}_{\alpha\beta}$  the metric components on the unit  $(n-1)$ -sphere.

We can now compute the Einstein tensor for the extended solution. Let's take  $G_{ij}^0 = \kappa^2 T_{ij}^0$ . Then we find that

$$\begin{aligned} G_{ij} &= G_{ij}^0 - \frac{1}{2} g_{ij} \frac{\hat{R}}{C^2} \\ &= \kappa^2 T_{ij}^0 - \frac{1}{2} g_{ij} \frac{(n-1)(n-2)}{C^2} \end{aligned} \quad (25)$$

$$\begin{aligned} G_{\alpha\beta} &= \hat{G}_{\alpha\beta} + \frac{\kappa^2}{q-2} g_{\alpha\beta} T^0 \\ &= \frac{\kappa^2}{q-2} g_{\alpha\beta} T^0 - \frac{1}{2} \frac{(n-2)(n-3)}{C^2} g_{\alpha\beta} \end{aligned} \quad (26)$$

where  $T^0 \equiv g^{ij} T_{ij}^0$ .

As we did in the previous section, we will show that the extra contribution can be obtained from a multiplet of  $n$  scalar fields in the hedgehog configuration.

Let's consider first the case in which the lower-dimensional metric is a solution to EFE's with a cosmological constant  $\Lambda^0$ :  $T_{ij}^0 = -\Lambda^0 g_{ij}$ . We assume that in the higher-dimensional case the energy-momentum tensor has contributions from a cosmological constant  $\Lambda$ , not necessarily the same as  $\Lambda^0$ , and the scalar fields. The contribution from the scalar fields is given by expression (3) with  $B^2(x^i) = C^2$  so that

$$T_{ij} \equiv -\Lambda g_{ij} - \frac{1}{2} (n-1) \frac{\eta^2}{C^2} g_{ij}, \quad (27)$$

$$T_{\alpha\beta} \equiv -\Lambda g_{\alpha\beta} - \frac{1}{2} (n-3) \frac{\eta^2}{C^2} g_{\alpha\beta}. \quad (28)$$

From equations (25), (26), (27) and (28), we see that the higher-dimensional EFE's reduce to

$$-\Lambda^0 - \frac{1}{2} \frac{(n-1)(n-2)}{C^2} = -\Lambda - \frac{n-1}{2} \frac{\kappa^2 \eta^2}{C^2} \quad (29)$$

$$-\frac{q\Lambda^0}{q-2} - \frac{1}{2} \frac{(n-2)(n-3)}{C^2} = -\Lambda - \frac{(n-3)}{2} \frac{\kappa^2 \eta^2}{C^2} \quad (30)$$

These can be solved for  $\Lambda$  and  $C^2$ . We must consider separately the cases  $\Lambda^0 = 0$  and  $\Lambda^0 \neq 0$ . In the latter case

$$\Lambda = \frac{q+n-3}{q-2} \Lambda^0, \quad C^2 = \frac{[(n-2) - \kappa^2 \eta^2](q+n-3)}{\Lambda}. \quad (31)$$

We see that for  $q > 2$ ,  $\Lambda$  has the same sign as  $\Lambda^0$ . On the other hand, since  $C^2 > 0$  it is clear that a solution with a positive (negative) cosmological constant is consistent only with a subcritical (supercritical) symmetry-breaking scale. The critical value is  $\eta_c^2 \equiv (n-2)\kappa^{-2}$ .  $n = 2$  is an exception to this, since in this case the solution exists only for a negative cosmological constant.

When  $\Lambda^0 = 0$ , the solution only exists when  $\eta = \eta_c$ . In this case  $\Lambda = 0$  and the value of  $C^2$  is arbitrary.

We can summarize the above by saying that if we want to compactify  $n-1$  extra dimensions on an  $(n-1)$ -sphere with a global defect, then we need  $\eta > \eta_c$  if the background cosmological constant is negative,  $\eta = \eta_c$ , when  $\Lambda = 0$  and  $\eta < \eta_c$  when the cosmological constant is positive.

The corresponding maximally symmetric geometries would be  $dS_q \times S^{n-1}$  when  $\Lambda > 0$ ,  $M_q \times S^{n-1}$  when  $\Lambda = 0$  and  $AdS_q \times S^{n-1}$  when  $\Lambda < 0$  ( $dS$  stands for deSitter,  $M$  for Minkowski and  $AdS$  for anti-de Sitter). This compactification scheme is in a way similar to that by Candelas and Weinberg [12]. They consider gravity and a set of massless scalar fields. The compactification arises from the quantum fluctuations of the fields while here it arises from the monopole-like configurations of classical fields, just like in [13] or [14] where gauge monopoles were considered.

It may seem surprising that we can have static solutions even in the case of vanishing cosmological constant. Since the curvature of the sphere acts as a cosmological constant in the transverse dimensions, one would not expect a static solution. However, we can see from the relations that this is possible when we introduce the hedgehog configuration for the scalar fields. In the transverse dimensions this also has the same effect as a cosmological constant and for the critical value of the symmetry-breaking scale its value cancels exactly the contribution from the  $(n-1)$ -sphere curvature.

Although the above has been derived for a cosmological constant, the result is more general. Consider now that  $G_{ij}^0 = \kappa^2 T_{ij}^0$  with  $T_{ij}^0 = -\Lambda^0 g_{ij} + S_{ij}^0$  so that in the lower-dimensional energy-momentum tensor we have contributions from a cosmological constant and some other form of energy which contributes through  $S_{ij}^0$ . In higher dimensions we add the corresponding term to the energy-momentum tensor

$$T_{ij} \equiv -\Lambda g_{ij} + S_{ij} - \frac{1}{2}(n-1) \frac{\eta^2}{C^2} g_{ij}, \quad (32)$$

$$T_{\alpha\beta} \equiv -\Lambda g_{\alpha\beta} + S_{\alpha\beta} - \frac{1}{2}(n-3) \frac{\eta^2}{C^2} g_{\alpha\beta} \quad (33)$$

This time we can find a solution to EFE's if  $\Lambda$  and  $C^2$  solve equations (29) and (30) as before, and if  $S_{ij} = S_{ij}^0$  and the angular components,  $S_{\alpha\beta}$ , satisfy the relation

$$S_{\alpha\beta} = g_{\alpha\beta}\bar{S}/(q-2) \quad \text{with} \quad \bar{S} \equiv g^{ij}S_{ij}^0. \quad (34)$$

Let's see that this is satisfied by massless scalar fields, which we represent collectively as  $\Psi$ . Our starting point is a configuration that solves Einstein's equations in  $q$  dimensions, so the fields are only functions of the coordinates  $\{x^i\}$ . In the higher-dimensional solution with the global charge we maintain the same ansatz and thus

$$S_{ij} = -\frac{1}{2}g_{ij}\partial_k\Psi\partial^k\Psi + \partial_i\Psi\partial_j\Psi \quad (35)$$

$$\bar{S} = -\frac{q-2}{2}\partial_k\Psi\partial^k\Psi \quad (36)$$

$$S_{\alpha\beta} = -\frac{1}{2}g_{ij}\partial_k\Psi\partial^k\Psi \quad (37)$$

From equations (36) and (37) we can indeed see that the relation  $S_{\alpha\beta} = g_{\alpha\beta}\bar{S}/(q-2)$  is satisfied.

One could also consider other sources, but then, this compactification scheme would only work if supplemented by the condition (34), which seems rather arbitrary. If we allow such arbitrariness, we could consider the compactification of a FRW-type solution. To construct the solution in  $q + (n - 1)$  dimensions with  $n - 1 > 2$  compactified extra dimensions, we start with a cosmological FRW solution with cosmological constant in  $q$  dimensions. We can embed this solution into  $n - 1$  extra dimensions if we consider an inhomogeneous fluid with

$$S_0^0 = -\rho(t), \quad S_J^I = \delta_J^I p(t), \quad S_\beta^\alpha = \delta_\beta^\alpha \bar{p}(t), \quad \text{with} \quad \bar{p}(t) \equiv \frac{(q-1)p(t) - \rho(t)}{q-2}, \quad (38)$$

where  $\bar{p}$  is defined in such a way as to satisfy condition (34). Here we have split the  $\{x^i\}$  coordinates as  $\{x^0, x^I\}$ , the 0 index referring to the time coordinate and capital latin letters  $I$  and  $J$  to the spatial coordinates.

The metric would be

$$ds^2 = -dt^2 + a^2(t) \left\{ \frac{dr^2}{1+kr^2} + r^2 d\Omega_2^2 \right\} + C^2 d\Omega_{n-1}^2 \quad (39)$$

with  $C^2$  given by (31).

By the same token, one can also consider the braneworld solutions in codimension one. In this models our universe is pictured as a domain wall and, in a bulk based approach, solutions can be obtained by considering a fixed background spacetime and studying the moving wall trajectories [15]. With the procedure described previously we can get solutions in which this domain wall has  $n - 1 > 2$  compactified extra dimensions. This gives explicit examples of solutions in which although we have a number of extra dimensions greater than one, only one can be considered large.

Considering for simplicity the  $Z_2$  symmetric walls as an example, we can write the metric for the background spacetime

$$ds^2 = -h(r)dt^2 + h^{-1}(r)dr^2 + r^2 \left[ \frac{d\chi^2}{1-k\chi^2} + \chi^2 d\Omega_2^2 \right] + C^2 d\Omega_{n-1}^2. \quad (40)$$

See [15] for details on the form of  $h(r)$ .

The position of the wall can be parameterized by  $\tau$ , the proper time of an observer comoving with the wall  $r = R(\tau)$ ,  $t = T(\tau)$ , where the trajectory is ultimately determined by the matter on the wall. For a given trajectory in the lower dimensional solution there will be a corresponding higher-dimensional one. In the original solution there is a cosmological constant in the bulk and a localized perfect fluid on the wall. In the higher-dimensional generalization the bulk has an additional contribution from the scalar fields and localized on the brane, which now has  $n - 1$  compactified extra dimensions, we have an inhomogeneous perfect fluid as that of equation (38) in order to satisfy the condition given by (34).

## V. CONCLUSIONS

In this paper we have investigated the gravitational effect of  $n$  scalar fields with the simplest nontrivial topological configuration, a radial configuration such that  $\vec{\phi} = \eta \hat{r}$ , where  $\hat{r}$  is the unit vector on the  $(n - 1)$ -sphere and  $\eta$  the symmetry-breaking scale.

As it is well known, such configurations can give rise to global defects. Here we have given the general form of the gravitational field, far from the core, of a global defect in the hedgehog configuration in a  $O(n)$  invariant solution. We have found that it amounts to the introduction of a deficit solid angle. Thus we can easily add the gravitational field of such a defect to any solution to Einstein's equations with  $O(n)$  invariance by adding the corresponding solid angle deficit which appears through an extra  $1 - (\kappa^2 \eta^2 / (n - 2))$  factor in the  $O(n)$  invariant part of the metric.

We have also considered the case where there is no defect because the nontrivial vacuum configuration can be maintained everywhere, as for textures. We have seen that when  $n$  scalar fields are in the hedgehog configuration this can lead to compactification of  $n - 1$  spatial dimensions. In particular, we have seen how the presence of the  $n$  scalar fields can be used to add  $n - 1$  compact spatial dimensions to any solution of Einstein's equations with a cosmological constant and massless scalar fields.

In the first case we have written down new solutions describing a global monopole in an expanding universe as well as black holes and black  $p$ -branes with a global charge. In the second case we have given explicit solutions where we add  $n - 1$  compact dimensions to known braneworld solutions in which our world is seen as a moving domain wall in a static bulk.

## VI. ACKNOWLEDGMENTS

I am grateful to Jose Juan Blanco-Pillado, Gia Dvali and Alexander Vilenkin for helpful discussions. This work was supported by the Basque Government under fellowship number BFI.99.89.

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